ASPEN ACADEMY STAGES OF MATHEMATICAL DEVELOPMENT WITHIN EACH UNIT OF STUDY

Each grade level will focus on mastery of 10-15 units of study per year. Each unit will begin with a pre-test. If the student achieves a 94% on that unit pre-test, they are moved out of the foundational stage entirely for that unit. Students scoring below a 94% will participate in the foundational exercises they demonstrated need for instruction in. In this way, we are truly teaching from a child's point of performance and moving them forward. Students entering into the Investigative and Experiential Stage and the Project Based Inquiry Stage will have curriculum, problems and projects that range from on grade level to the 12th grade level depending on their ability levels.

Teaching to Mastery: Each topic is covered in detail and taught to mastery. Immediately after new concepts are taught, students are engaged with a variety of mathematically rich problems. This ensures that the focus is on the student's deep understanding of each topic. Singapore Math is geared towards producing mathematical thinkers, and it does this by walking children through all the component parts of a problem before presenting them with the whole problem to solve.

Spiral Progression: Topics covered previously are reviewed at higher grades and with increasing difficulty. The introduction of new concepts is built upon the mathematical concepts students have learned previously. Spiral progression also allows for a review of important math concepts while expanding on that foundation.

Meta-cognition: Meta-cognition refers to the ability to monitor one's own thought processes. In teaching students to be conscious of the strategies they use to accomplish a task, this strategy encourages students to think of alternative means of solving problems and promotes logical thinking. Students are encouraged to be aware of how they arrive at their solutions. Alternative ways of solving the problem are provided as a form of guidance for students to check their thought processes. This is opposed to rote learning and application of formulaic strategies.

Pedagogical Approach and Methodology
Concrete – Pictorial – Abstract Approach
This approach enables students to encounter math in a meaningful way and translate mathematical skills from the concrete to the abstract. This approach allows students to understand mathematical concepts before learning the "rules" or formulaic expressions:

1. Students first encounter the mathematical concepts through the use of manipulatives.
2. Students then move on to the pictorial stage in which pictures are used to model problems.
3. When students are familiar with the ideas taught, they progress to a more advanced or abstract stage in which only numbers, notation and symbols are used.

"Students experience the concrete, pictorial or abstract levels of mathematics in each unit. The concrete stage is the stage used to introduce a new topic. Kinesthetic learners love the concrete stage, because it involves hands-on manipulatives which the students will use to begin the understanding of a new concept. Manipulatives can include numerous things, including counters, (mostly used at the primary level), base ten blocks, centimeter cubes, unifix cubes, blocks, etc. At the concrete stage students may also act out a problem, which helps with understanding and retention. After the concept has been introduced the students usually move to the textbook/workbook, where they will start to work at the pictorial stage. In the textbook, the pictures are often provided for the students. Starting in third level books, the students learn to draw their own "pictures" known as bar models, and continue to use these models through the sixth level. The abstract level is the highest level. And is the situation where the student uses symbols to solve problems. Students in the third level books will spend most of their school year learning to draw the bar models correctly. When they've mastered drawing the bar models for a concept, the teacher will allow them to use symbols to solve problems and to create

problems of their own. The theory in Singapore Math is that the abstract level should not be obtained in a topic until the students have mastered the use of concrete materials and viewed/drawn pictures to solve similar problems."

Tricia Salerno, Master Teacher, Mathematics and National Singapore Math Trainer

Foundational Stage
Concrete Stage
In this Foundational Stage, students are taught math concepts through direct instruction, discussion, and use of manipulatives and specific application of newly learned skills. Here, they learn how to answer questions related to the topic using the workbooks and textbooks. These texts are designed to present a mathematical concept in multiple ways, facilitating the function of deducing solutions in a myriad of fashions. Additionally, this stage also promotes automaticity and mastery of math facts, essential to have for a child before moving into higher order mathematics. This stage is easily assessed and students are accountable for mastery before moving into the Investigative and Experiential Stage and the Project Based Inquiry Stage.

Investigative and Experiential Stage
Pictorial Stage
The Investigative and Experiential Stage is designed to help students apply and analyze topics learned in the Foundational Stage, but in a different, two-dimensional application. Here, students may spend a long period of time on a challenging word problem that uses the same skills taught earlier in the unit, but requires more problem-solving heuristics. Students will work on materials that are written in a different style than Singapore math to help build test-taking, deductive reasoning and comprehension skills. They may take concepts learned through the workbook and extend it to an activity that requires a more complicated use of manipulatives or visual representations. For instance, a student who learned the factors of 12 may be asked in the pictorial stage to show the factors of 12, 24, and 96 with unit cubes in arrays. The teacher can observe their learning through the articulation of building, drawing (graph paper) and writing (a paragraph summary of their thinking). Model drawing is an ingenious problem solving strategy built into the Primary Mathematics curriculum. Students are taught to visualize and construct concrete pictures to help them make sense of word problems. The model drawing method requires students to understand mathematical concepts underlying word problems and equips them with a strong conceptual foundation in mathematics to solve even the most challenging problems. The model drawing technique not only provides a powerful method for solving problems, but also serves as a link to algebra. Symbolic representation of problems, the mainstay of algebra, emerges as a logical extension of the model drawing technique.

INVESTIGATIVE AND EXPERIENTIAL STAGE: ALGEBRA EXAMPLES

Into the Unknown
Working in whole class, pairs and individual situations, students explore the "unknowns" of algebra as they take a simulated trip into the sea on a submarine. Each phase takes the sub deeper into the ocean, culminating with students building an underwater research station using their new skills. Students will:

- Investigate variables, equations, patterns and functions
- Connect algebra skills with concrete materials through problem-solving situations
- Write daily in "Water Logs" to explain their thinking
- Display mathematical relationships through symbols, diagrams, tables and graphs

Algebra Mystery Maze

Students compete in teams to solve algebra problems quickly and design mazes that challenge their classmates. Students will:

- Solve algebraic equations and simple algebra word problems
- Develop confidence to work algebra problems quickly and accurately
- Apply skills of measurement to navigate and design mazes
- Apply math communication skills to describe problem solving strategies
- Work cooperatively as they rotate through the roles of Measurer, Designer and Recorder

The Project Based Inquiry Stage
Abstract Stage
The project based inquiry stage is based on the student's need to synthesize and evaluate their mathematical knowledge in the context of real-world and abstract application. Here, students are involved in units of study structured around a complex central problem. Although, each unit has a specific mathematical focus, other topics are brought in as needed to solve the central problem, rather than narrowly restricting the mathematical content. Ideas that are developed in one unit can be revisited and deepened in later units. This project based inquiry stage was designed in response to a critical deficiency in our country. As students enter the professions and trades, demands will be placed on them that focus on their problem-solving and communication skills. Preparing students for the challenges of business and industry requires a shift in instruction away from routine manipulation of symbols and procedures toward an in depth, conceptual understanding of mathematics. At this stage, students will become familiar with how math is applied in the real world and become comfortable recognizing math at this level.

EXAMPLES OF FOUNDATIONAL, INVESTIGATIVE AND EXPERIENTIAL STAGE AND PROJECT BASED INQUIRY STAGES

Testimony of John Hoven
On Behalf of The Center for Education Reform
At the National Public Forum on the Draft 2004 Mathematics Framework
September 24, 2001

Good morning. My name is John Hoven, a member of, and speaking on behalf of the Center for Education Reform. The Center is a national, independent, non-profit advocacy organization founded in 1993 to provide support and guidance to parents and teachers, community and civic groups, policymakers and grassroots leaders, and all who are working to bring fundamental reforms to their schools.

By way of credentials, I am a Ph.D economist, with a bachelor's degree in math and physics and a master's degree in physics. Currently, I serve as co-president of the Gifted and Talented Association of Montgomery County, Maryland. The Center for Education Reform appreciates your invitation to offer recommendations on the Draft 2004 Mathematics Framework for the National Assessment of Educational Progress (NAEP).

My remarks focus on the question of whether the mathematics objectives in the draft framework are appropriate expectations for students in grades 4, 8, and 12. We believe they are not, for they do not depart substantially from current trends in the exam.

My remarks are based on an examination of the publicly released NAEP Test Questions (http://nces.ed.gov/nationsreportcard/itmrls/). I compared these with a published set of exam problems based on the math curriculum of Singapore.1 I chose Singapore because its TIMSS scores (Third International Math and Science Study) make it the acknowledged world leader in mathematics. My point is simple: There is a chasm of difference in *expectations between NAEP and the problems used by world-class mathematics leaders. We expect too little from our children, and by lowering our expectations we lower their incentive to achieve.*

As an example, my own school district – Montgomery County, Maryland – is one of the most affluent, highly educated counties in America, yet our gifted students scored at the level of Singapore's average student NAEP classifies its problems as "easy," "medium," or "hard." I benchmarked the "hard" 8th grade problems, examining NAEP's highest level of expectation for 8th grade math. Most of these "hard" 8th grade problems are at the level of Singapore's grade 5 –or lower. I want you to see for yourself some of these "hard" 8th grade problems.

Consider: In one problem, for example, the student is shown a "Lunch Menu" with items like Onion Soup for $.80 and Ice Cream for $1.10. The question asks: "What is the total cost of Soup of the Day, Beefburger with Fries, and Cola?"

This is considered a "hard" eighth grade problem. But Singapore has harder problems than this in grade 3. Here are two examples:

1) 5 oranges cost $2.25. What is the cost of 12 oranges? ____________

2) I want to buy a calculator for $29.70 and a watch for $32.00. I have $28.50. How much more money do I need?
(1) $26.20
(2) $30.80
(3) $33.20
(4) $32.70

Both of these are two-step math problems. They illustrate Singapore's expectation that all children should acquire mastery of the math skills needed for algebra and beyond. NAEP's expectation is that children need to be able to order take-out from McDonald's.

NAEP's "hard" 8th grade problems are mostly at the level of Singapore's 5th grade problems.

FOUNDATIONAL LEVEL: ALGEBRA EXAMPLES

In 8th grade, mathematically advanced American students take Algebra 1. NAEP ignores all of these children. Not a single question is at an Algebra I level. Here is NAEP's most difficult algebra question:

3) The length of a rectangle is 3 more than its width. If L represents the length, what is an expression for the width?
A) 3 ÷ L
B) L ÷ 3
C) L x 3
D) L + 3
E) L - 3

Frankly, this kind of problem is for a child who started learning algebra yesterday. By comparison, here is a Singapore problem for grade 6:

4) Ahmad scored x marks for his English in an examination. He scored 90, 80, 80 for 3 of his other 4 subjects, and did half as well in English as he did in Maths. If he had an overall average of 80 marks, how many marks did he score for Maths?
Answer: ____________

Why does NAEP expect so little, and Singapore expect so much? Because Singapore students have been solving progressively more complex problems since third grade.

The two Singapore algebra problems below further illustrate the process of building math skills step by step. The first problem is the distributive law. Singapore students master this in 6th grade, so they can use it automatically in 8th grade Algebra.

5) 6p(3 + 5p) is the same as ________________.
(1) 18p + 30p2
(2) 18p + 3
(3) 11p2 + 30p
(4) 11p + 3p2

The second problem begins the process of "chunking" – learning to see algebraic expressions as a single "chunk." That skill makes it easy to see that you just divide $20 by p2 kg to get $20/p2.

6) If p2 kg of rambutans cost $20, what is the cost of 1 kg of rambutans?
(1) $20 - p
(2) $20 + p
(3) $20/p
(4) $20/p2

Finally, here are two more problems demonstrating that by 6th grade, algebra is a useful tool for Singapore children. Here they apply it to geometry and percents:

7) Rama bought n sets of toys and 6 pairs of shirts for $65. Each shirt costs $5.
(a) Find the cost of each set of toys in terms of n.
(b) If Rama received a 20% discount for all the items, how many more sets of toys could she buy if n = 5 for part (a).

In comparison, NAEP doesn't test algebra; it tests "algebraic concepts."

Conclusion

Whereas 60 years ago, good-paying jobs existed for those without math skills and the ability to compute (and read), the world-class competitiveness of tomorrow demands that the next generation of children be prepared to meet more extensive challenges than we have ever witnessed before.

To do less is to fail them, and fail our nation.
We believe that the 2004 NAEP be required to meet world-class standards. This means that NAEP's easy problems must be similar to easy problems on exams in Singapore, Japan, or some similar world-class math power, and NAEP's hard problems must be similar to their hard problems.
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Answers to problems:
1) $5.40 for 12 oranges
2) I need $33.20 more
3) E) width = L-3
4) 100 on Maths
5) 5p(3 + 5p) = 18p + 30p2
6) $20/p2
7) (a) $35/n, (b) 2 more sets of toys

THE PROJECT BASED INQUIRY STAGE: ALGEBRAIC EXAMPLES

Game Factory
Cheatum Swindle is running the Goodwin's game factory into the ground by producing unfair games, and it's up to your students to use their arithmetic skills to save the company! Students work in pairs performing hands-on experiments with spinners, dice, coins and cards to test the probabilities of Cheatum's games. The flip of a coin or the roll of the die determines the moves they make as they advance through the factory examining games for fairness. As they find problems, they make modifications and record reasons for their decisions. In the final push to save the company's reputation, student pairs design their own games and present them with an explanation of their fairness.

The Overland Trail
This unit looks at the mid-nineteenth century western migration across the United States in terms of the many linear relationships involved. These relationships grow out of the study of planning what to take on the 2,400-mile trek, estimating the cost of the move, and studying rates of consumption and of travel.

Students construct mathematical models and draw graphs by hand and with a graphing calculator. They interpret graphs in terms of the "stories" the graphs tell, and create graphs from "stories." They write algebraic expressions that represent situations, use manipulatives to represent variables, and solve systems of equations using graphs made by hand and by a function-graphing facility on a calculator. In the process of graphing equations, they see the need to solve equations for one variable in terms of another, and learn techniques for doing so.

Solve It!
This unit focuses on using equations to represent real-life situations and on developing the skills to solve these equations. Students begin with situations used in the first year of the curriculum and develop algebraic representations of problems. To find solutions to the equations that arise, students explore the concepts of equivalent expressions and equivalent equations.

Using these concepts, they learn principles such as the distributive property for working with algebraic expressions and equations, and acquire methods that they can use to solve any linear equation. They also explore the relationships among an algebraic expression, a function, an equation, and a graph, and examine ways to use graphs to solve nonlinear equations.

Fireworks
The central problem of this unit involves sending up rockets to create a fireworks display. The trajectory of the rocket is a parabola; this unit continues the algebraic investigations of Solve It! with a special focus on quadratic expressions, equations, and functions. Students see that they can use algebra to find the vertex of the graph of a quadratic function by writing the quadratic expression in a particular form.

The Urns of King Midas (*Abstract Example*)
King Midas made 4 Urns to carry his gold bars. For good luck he wrote a 4 digit number - one digit on each urn. The number he chose was special: 1210. The "0 urn" contained the number of zeros in the number. The "1 urn" contained the number of ones in the number. The "2 urn" contained the number of twos in the number, and the "3 urn" contained the number of threes in the number

For example, the "1 urn" has a 2 written on it which means that the number 1210 must have two ones.
Later King Midas made five urns. For good luck, what number should he put on them?

Extensions: What about even more urns? Are there an infinite number of solutions?

ASSESSMENTS

Aspen Academy students, on average, placed in the top 8 percent in the nation in honors standardized testing. Each class has an aggregate performance that places them 2-5 grade levels above their national peers in Grade Mean Equivalent standards.

PARENT UNIVERSITY MATH TRAINING SESSIONS

Parents will have the opportunity to be trained in the methods applied in Singapore mathematics throughout the year so that they can be a great resource to help students thrive in the classroom. Parent mathematics training will take place on a monthly base and be posted on the regular calendar